Algorithm for Runge – Kutta Method of order 2

⇒ Suppose we want to find an approximate solution of the order differential equation.
⇒ dy/ dx = f(x,y) with y(x₀) = y₀
⇒ Then algorithm for Runge – Kutta method of order 2 is given as
⇒ Step 1: Define f(x,y), x₀, y₀ and xn.
⇒ Step 2: Find k by using \( h = \frac{(xₙ - x₀)}{n} \)
⇒ Step 3: Find \( K₁ \), \( K₂ \) and hence K by using
  \[
  K₁ = hf(x₀,y₀) \\
  K₂ = hf(x₀+h,y₀+k₁) \\
  K = \frac{1}{2}(K₁ + K₂)
  \]
⇒ Step 4: Find \( y₁ = y₀ + K \)
⇒ Step 5: Repeat step 3 for successive approximation by using
  \[
  K₁ = hf(xᵢ,yᵢ) \\
  K₂ = hf(xᵢ+h,yᵢ+K₁/2) \\
  K₃ = hf(xᵢ+h,yᵢ+K') \text{ where } K' = hf(xᵢ+h,yᵢ+k₁) \\
  K = \frac{1}{6}(K₁ + 4K₂ + K₃)
  \]
⇒ Step 6: \( yᵢ₊₁ = yᵢ + K \)
⇒ Step 7: End

Algorithm for Runge – Kutta Method of order 3

⇒ Suppose we want to find an approximate solution of the order differential equation.
⇒ dy/ dx = f(x,y) with y(x₀) = y₀
⇒ Then algorithm for Runge – Kutta method of order 3 is given as
⇒ Step 1: Define f(x,y), x₀, y₀ and xn.
⇒ Step 2: Find by using \( h = \frac{(xₙ - x₀)}{n} \)
⇒ Step 3: Find \( K₁ \), \( K₂ \) and hence K by using
  \[
  K₁ = hf(x₀,y₀) \\
  K₂ = hf(x₀+h/2,y₀+K₁/2) \\
  K₃ = hf(x₀+h,y₀+K') \text{ where } K' = hf(x₀+h,y₀+k₁) \\
  K = \frac{1}{6}(K₁ + 4K₂ + K₃)
  \]
⇒ Step 4: Find \( y₁ = y₀ + k \)
⇒ Step 5: Repeat step 3 for successive approximation by using
K_1 = hf(x_i, y_i)
K_2 = hf(x_i + h/2, y_i + K_1/2)
K_3 = hf(x_i + h, y_i + K') where
K' = hf(x_i + h, y_i + k_1)
K = 1/6 (K_1 + 4K_2 + K_3)
Where I = 1, 2, 3, ...

⇒ Step 6: Find y_{i+1} = y_i + K
⇒ Step 7: End

Algorithm for Runge – Kutta Method of order 4

Suppose we want to find an approximate solution of the order differential equation.
⇒ dy/ dx = f(x, y) with y(x_0) = y_0
⇒ Then algorithm for Runge – Kutta method of order 4 is given as
⇒ Step 1: Define f(x, y), x_0, y_0 and x_n.
⇒ Step 2: Find by using h = (x_n – x_0)/n
⇒ Step 3: Find K_1, K_2, K_3, K_4 and hence K by using
⇒ K_1 = hf(x, y)
⇒ K_2 = h(f(x_0 + h/2, y_0 + k_1/2))
⇒ K_3 = hf(x_0 + h/2, y_0 + K_2/2)
⇒ K_4 = hf(x_0 + h, y_0 + K_3)
⇒ K = 1/6(K_1 + 2K_2 + 2K_3 + K_4)
⇒ Step 4: Find y_1 = y_0 + k
⇒ Step 5: Repeat step 3 for successive approximation by using
⇒ K_1 = hf(x, y)
⇒ K_2 = h(f(x_i + h/2, y_i + k_i/2))
⇒ K_3 = hf(x_i + h/2, y_i + K_2/2)
⇒ K_4 = hf(x_i + h, y_i + K_3)
⇒ K = 1/6(K_1 + 2K_2 + 2K_3 + K_4)
⇒ Where I = 1, 2, 3, ...
⇒ Step 6: y_{i+1} = y_i + K
⇒ Step 7: End
Algorithm for Taylor’s series

⇒ Step 1: Define \( f(x,y) \), \( x_0 \), \( y_0 \) and \( x_n \)
⇒ Step 2: If we need the values of \( y \) at \( n \) points besides \( x_0 \) in \([x_0,x_n]\) which are equally spaced, then we choose the step length as \( h = (x_n - x_0)/n \) so what
\[
\begin{align*}
x_n &= x_0 + nh \\
y_n &= y(x_0 + nh) \ 	ext{with} \ I = 1, 2, 3, \ldots
\end{align*}
\]
⇒ Step 3: Find the approximations \( y \) to the exact values \( y(x_n) \) by the recursive formula
\[
y_{i+1} = y_i + hT_j(x_i, y_i) \quad I = 0, 1, 2
\]
\[
T_j(x,y) = f(x,y) + h/2!f'(x,y) + \ldots + h^{j-1}/j!f^{j-1}(x,y) \ 	ext{for} \ j = 1, 2, 3 \ldots
\]
⇒ Step 4: Write the solutions \( y_n \) for \( x = x_n \)

Algorithm for Milne’s Predictor Corrector Method

⇒ Step 1: Input four values of \( x \) and \( y \) as \( x_0 \), \( x_1 \), \( x_2 \), \( x_3 \) and \( y_0 \), \( y_1 \), \( y_2 \), \( y_3 \). Also input this number of subintervals \( n \).
⇒ Step 2: Calculate \( h = x_1 - x_0 \)
⇒ Step 3: Input \( f(x,y) \)
⇒ Step 4: Calculate \( f(x_0, y_0) \), \( f(x_1, y_1) \), \( f(x_2, y_2) \) and \( f(x_3, y_3) \)
⇒ Step 5: Apply predictor’s formula
\[
y_4 = y_0 + 4h/3 \left[ 2f(x_1, y_1) - f(x_2, y_2) + 2f(x_3, y_3) \right]
\]
⇒ Step 6: Calculate \( f(x_4, y_4) \)
⇒ Step 7: Apply corrector formula
\[
y^c_4 = y_2 + h/3\left[ f(x_2, y_2) + 4f(x_3, y_3) + f(x_4, y_4) \right]
\]
⇒ Step 8: Again apply corrector formula till you get the difference between any two corrected values less than the predicted accuracy
⇒ Step 9: Continue this process till you get the result.
Algorithm for Adams Bash forth Method

⇒ **Step 1:** Input four values x and y as \(x_0, x_{-1}, x_{-2}, x_{-3}\) and \(y_0, y_{-1}, y_{-2}, y_{-3}\). Also input the number of sub intervals.
⇒ **Step 2:** Calculate \(h = x_0 - x_{-1}\)
⇒ **Step 3:** Input \(f(x, y)\)
⇒ **Step 4:** Calculate \(f_0 = f(x_0, y_0)\)

\[
\begin{align*}
  f_{-1} &= f(x_{-1}, y_{-1}) \\
  f_{-2} &= f(x_{-2}, y_{-2}) \\
  f_{-3} &= f(x_{-3}, y_{-3})
\end{align*}
\]

⇒ **Step 5:** Apply predictor formula:

\[
y_1 = y_0 + \frac{h}{24}(55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3})
\]
⇒ **Step 6:** Calculate \(f(x_1, y_1)\)
⇒ **Step 7:** Apply corrector formula:

\[
y_{1c} = y_0 + \frac{h}{24}(9f_1 + 19f_0 - 5f_{-1} + f_{-2})
\]
⇒ **Step 8:** Again apply corrector formula till you get the difference between any two corrected values less than the predicated accuracy
⇒ **Step 9:** Continue this process till you get the result.